MATH 181E: Mathematical Statistics - Time Series
Discussion 7:

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Outline

Gauss-Newton procedure

Back to the MA(1) process, $x_t = w_t + \theta w_{t-1}$. Write the truncated errors as

$$w_t(\theta) = x_t - \theta w_{t-1}(\theta), \quad t = 1, \dots, n,$$

where we condition on $w_0(\theta) = 0$. Taking derivatives and negating,

$$-\frac{\partial w_t(\theta)}{\partial \theta} = w_{t-1}(\theta) + \theta \frac{\partial w_{t-1}(\theta)}{\partial \theta}, \quad t = 1, \dots, n,$$

where $\partial w_0(\theta)/\partial\theta = 0$.
If we plug $z_t(\theta) = -\partial w_t(\theta)/\partial\theta$ and $z_0(\theta) = 0$ into the formula, we can get

$$z_t(\theta) = w_{t-1}(\theta) - \theta z_{t-1}(\theta), \quad t = 1, \dots, n,$$

Gauss-Newton procedure

Let $\theta_{(0)}$ be an initial estimate of θ.

$$\widehat{\beta - \beta_{(0)}} = \left(\sum_{t=p+1}^{n} z_t \left(\beta_{(0)}\right) z_t^\top \left(\beta_{(0)}\right) \right)^{-1} \left(\sum_{t=p+1}^{n} z_t \left(\beta_{(0)}\right) w_t \left(\beta_{(0)}\right) \right)$$

We can write the one-step Gauss-Newton estimate as

$$\beta_{(1)} = \beta_{(0)} + \widehat{\beta - \beta_{(0)}},$$

Then, $p = 0$ and the Gauss-Newton procedure for conditional least squares is given by

$$\theta_{(j+1)} = \theta_{(j)} + \frac{\sum_{t=1}^{n} z_t \left(\theta_{(j)}\right) w_t \left(\theta_{(j)}\right)}{\sum_{t=1}^{n} z_t^2 \left(\theta_{(j)}\right)}, \quad j = 0, 1, 2, \ldots,$$